ATTACHMENT A

BEACHFILL DIFFUSION CALCULATIONS

Date: July 18, 2013
Analyst: Rob Hampson, Moffatt & Nichol
Client: U.S. Army Corps of Engineers
Project: Fire Island Interim
Analysis: Alongshore Diffusivity

Solving for the Alongshore Diffusivity along Fire Island based on predicted Gross Sediment Transport Rate

Reference: Dean R. G., Dalyrmple R. A., 2002. "Coastal Processes with Engineering Applications," Cambridge University Press, New York, NY.)

Definitions

$GST := 2250000 \cdot \frac{m^3}{yr}$ — gross sediment transport at Fire Island (Gravens et al., 1999)

$S := 2.65$ — specific gravity of sand

$p := 0.35$ — porosity of sand

$\gamma b := 0.78$ — breaking wave index

$K := 0.77$ — sediment transport coefficient for medium sand (e.g. 0.3 mm) (Komar & Inman 1970)

$\theta b := 10 \cdot deg$ — effective breaking wave angle

$hc := 27 \cdot ft$ — depth of closure (NGVD)

$B := 9.5 \cdot ft$ — berm elevation (NGVD)

$Tp := 8 \cdot s$ — assumed effective wave period

CERC Sediment Transport Equation

$$GST = \blacksquare \cdot Cp \cdot Hb^{\frac{5}{2}} \cdot \sin(2 \cdot \theta b)$$ CERC Equation

$$Cp := K \cdot \frac{\sqrt{\frac{g}{\gamma b}}}{16 \cdot (S-1) \cdot (1-p)} = 0.159\, m^{0.5} \cdot s^{-1}$$

$$Hb := \left(\frac{GST}{Cp \cdot \sin(2 \cdot \theta b)}\right)^{\frac{2}{5}} = 1.114\, m$$ effective breaking wave height

Alongshore Diffusivity, G

$$G := 2 \cdot Cp \cdot Hb^{\frac{5}{2}} \cdot \frac{\cos(2 \cdot \theta b)}{hc + B} = 0.035\, m^2 \cdot s^{-1}$$

Effect of Wave Refraction on Alongshore Diffusivity, G

Reference: Dean R. G., 2005. "Advanced Series on Ocean Engineering - Volume 18: Beach Nourishment Theory and Practice," World Scientific Publishing Co., Hackensack, NJ.)

Dean showed that wave refraction at a beachfill project can reduce the alongshore diffusivity by the ratio Cb/Cc where Cb and Cc are the wave celerity at breaking and depth of closure respectively

Wave Length and Celerity at depth of closure

$L := 150 \cdot m$ Initial value

Given

$$L = \frac{g}{2 \cdot \pi} \cdot T^2 \cdot \tanh\left(2 \cdot \pi \cdot \frac{depth}{L}\right)$$

$Wavel(T, depth) := Find(L)$

$Lc := Wavel(Tp, hc) = 65.641\, m$

$Cc := \frac{Lc}{Tp} = 8.205\, m \cdot s^{-1}$

Wave Length and Celerity at break point

$L := 150 \cdot m$ Initial value

Given

$$L = \frac{g}{2 \cdot \pi} \cdot T^2 \cdot \tanh\left(2 \cdot \pi \cdot \frac{depth}{L}\right)$$

$Wavel(T, depth) := Find(L)$

$Lb := Wavel\left(Tp, \frac{Hb}{\gamma b}\right)$ $Lb = 29.492\, m$

$Cb := \frac{Lb}{Tp} = 3.687\, m \cdot s^{-1}$

$\frac{Cb}{Cc} = 0.449$

$ref := 0.4$ set reduction factor to 0.4

Adjusted Alongshore Diffusivity, G

$G := G \cdot ref = 0.014\, m^2 \cdot s^{-1}$

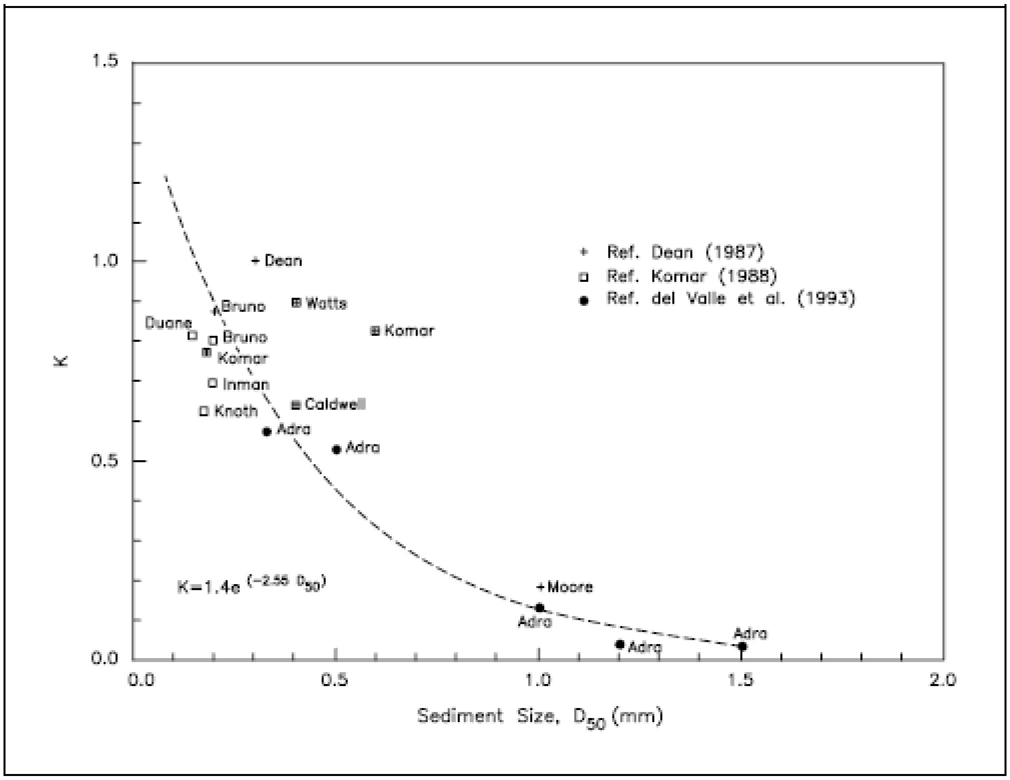

Figure III-2-6. Coefficient *K* versus median grain size D_{50} (del Valle, Medina, and Losada 1993)

Reproduced from CEM, Page III-2-15

Date: July 18, 2013
Analyst: Rob Hampson, Moffatt & Nichol
Client: U.S. Army Corps of Engineers
Project: Fire Island Interim
Analysis: Beachfill Diffusion - Western Fire Island - MIDU

Calculate fraction of beachfill volume remaining with Pelnard-Considere Equation for Rectangular Beachfill

Reference: Dean R. G., Dalyrmple R. A., 2002. "Coastal Processes with Engineering Applications," Cambridge University Press, New York, NY.)

Note: This analysis does not include beachfill tapers, the final analysis for the Fire Island Interim Project applied 6 degree berm tapers, which requires the trapazoidal beachfill solution to the Pelnard-Considere Equation. The trapzoidal beachfill solution was solved numerically in Matlab.

Definitions

$hc := 27 \cdot ft$ depth of closure (NGVD)

$B := 9.5 \cdot ft$ berm elevation (NGVD)

$G := 0.014 \cdot \frac{m^2}{s}$ alongshore diffusivity

$Yo := 50.5 \cdot ft$ initial cross-shore distance between MIDU design shoreline and natural shoreline

$Yb := 0 \cdot ft$ offset between MIDU baseline and evaluated baseline

$Ya := 20 \cdot ft$ advance nourishment width

$l := 41800 \cdot ft$ alongshore length of beachfill

$t := 4yr$ time after initial placement

$be := 3 \cdot \frac{ft}{yr}$ background erosion rate

Analytical Solution to Pelnard-Considere for Rectangular Beachfill Project

$$M := \left[\frac{\sqrt{4 \cdot G \cdot t}}{l \cdot \sqrt{\pi}} \cdot \left[e^{-\left(\frac{l}{\sqrt{4 \cdot G \cdot t}}\right)^2} - 1\right] + \mathrm{erf}\left(\frac{l}{\sqrt{4G \cdot t}}\right)\right] - \frac{be \cdot t}{(Yo + Yb + Ya)}$$

fraction of initial fill remaining after time t

$M = 0.712$

$re := (1 - M) \cdot \frac{(Yo + Yb + Ya)}{t} = 5.075 \cdot \frac{ft}{yr}$ representative erosion rate

$de := re - be = 2.075 \cdot \frac{ft}{yr}$ diffusive erosion rate

Date: July 18, 2013
Analyst: Rob Hampson, Moffatt & Nichol
Client: U.S. Army Corps of Engineers
Project: Fire Island Interim
Analysis: Beachfill Diffusion - Western Fire Island - MREI

Calculate fraction of beachfill volume remaining with Pelnard-Considere Equation for Rectangular Beachfill

Reference: Dean R. G., Dalyrmple R. A., 2002. "Coastal Processes with Engineering Applications," Cambridge University Press, New York, NY.)

Note: This analysis does not include beachfill tapers, the final analysis for the Fire Island Interim Project applied 6 degree berm tapers, which requires the trapazoidal beachfill solution to the Pelnard-Considere Equation. The trapzoidal beachfill solution was solved numerically in Matlab.

Definitions

$hc := 27\cdot ft$	depth of closure (NGVD)
$B := 9.5\cdot ft$	berm elevation (NGVD)
$G := 0.014\cdot \frac{m^2}{s}$	alongshore diffusivity
$Yo := 50.5\cdot ft$	initial cross-shore distance between MIDU design shoreline and natural shoreline
$Yb := 34.80\cdot ft$	offset between MIDU baseline and evaluated baseline
$Ya := 24.3\cdot ft$	advance nourishment width
$l := 41800\cdot ft$	alongshore length of beachfill
$t := 4yr$	time after initial placement
$be := 3\cdot \frac{ft}{yr}$	background erosion rate

Analytical Solution to Pelnard-Considere for Rectangular Beachfill Project

$$M := \left[\frac{\sqrt{4\cdot G\cdot t}}{l\cdot\sqrt{\pi}}\cdot\left[e^{-\left(\frac{l}{\sqrt{4\cdot G\cdot t}}\right)^2} - 1\right] + \text{erf}\left(\frac{l}{\sqrt{4G\cdot t}}\right)\right] - \frac{be\cdot t}{(Yo + Yb + Ya)}$$

fraction of initial fill remaining after time t

$M = 0.773$

$$re := (1 - M)\cdot\frac{(Yo + Yb + Ya)}{t} = 6.226\cdot\frac{ft}{yr}$$

representative erosion rate

$$de := re - be = 3.226\cdot\frac{ft}{yr}$$

diffusive erosion rate

Date: July 18, 2013
Analyst: Rob Hampson, Moffatt & Nichol
Client: U.S. Army Corps of Engineers
Project: Fire Island Interim
Analysis: Beachfill Diffusion - Fire Island Pines - MIDU

Calculate fraction of beachfill volume remaining with Pelnard-Considere Equation for Rectangular Beachfill

Reference: Dean R. G., Dalyrmple R. A., 2002. "Coastal Processes with Engineering Applications," Cambridge University Press, New York, NY.)

Note: This analysis does not include beachfill tapers, the final analysis for the Fire Island Interim Project applied 6 degree berm tapers, which requires the trapazoidal beachfill solution to the Pelnard-Considere Equation. The trapzoidal beachfill solution was solved numerically in Matlab.

Definitions

$hc := 27 \cdot ft$	depth of closure (NGVD)
$B := 9.5 \cdot ft$	berm elevation (NGVD)
$G := 0.014 \cdot \frac{m^2}{s}$	alongshore diffusivity
$Yo := 28.2 \cdot ft$	initial cross-shore distance between MIDU design shoreline and natural shoreline
$Yb := 0 \cdot ft$	offset between MIDU baseline and evaluated baseline
$Ya := 40 \cdot ft$	advance nourishment width
$l := 6400 \cdot ft$	alongshore length of beachfill
$t := 4yr$	time after initial placement
$be := 0 \cdot \frac{ft}{yr}$	background erosion rate

Analytical Solution to Pelnard-Considere for Rectangular Beachfill Project

$$M := \left[\frac{\sqrt{4 \cdot G \cdot t}}{l \cdot \sqrt{\pi}} \cdot \left[e^{-\left(\frac{l}{\sqrt{4 \cdot G \cdot t}}\right)^2} - 1\right] + \operatorname{erf}\left(\frac{l}{\sqrt{4G \cdot t}}\right)\right] - \frac{be \cdot t}{(Yo + Yb + Ya)}$$

fraction of initial fill remaining after time t

$M = 0.38$

$$re := (1 - M) \cdot \frac{(Yo + Yb + Ya)}{t} = 10.563 \cdot \frac{ft}{yr}$$

representative erosion rate

$$de := re - be = 10.563 \cdot \frac{ft}{yr}$$

diffusive erosion rate

Date: July 18, 2013
Analyst: Rob Hampson, Moffatt & Nichol
Client: U.S. Army Corps of Engineers
Project: Fire Island Interim
Analysis: Beachfill Diffusion - Fire Island Pines - MREI

Calculate fraction of beachfill volume remaining with Pelnard-Considere Equation for Rectangular Beachfill

Reference: Dean R. G., Dalyrmple R. A., 2002. "Coastal Processes with Engineering Applications," Cambridge University Press, New York, NY.)

Note: This analysis does not include beachfill tapers, the final analysis for the Fire Island Interim Project applied 6 degree berm tapers, which requires the trapazoidal beachfill solution to the Pelnard-Considere Equation. The trapzoidal beachfill solution was solved numerically in Matlab.

Definitions

$hc := 27 \cdot ft$	depth of closure (NGVD)
$B := 9.5 \cdot ft$	berm elevation (NGVD)
$G := 0.014 \cdot \frac{m^2}{s}$	alongshore diffusivity
$Yo := 28.2 \cdot ft$	initial cross-shore distance between MIDU design shoreline and natural shoreline
$Yb := 34.4 \cdot ft$	offset between MIDU baseline and evaluated baseline
$Ya := 77.1 \cdot ft$	advance nourishment width
$l := 6400 \cdot ft$	alongshore length of beachfill
$t := 4yr$	time after initial placement
$be := 0 \cdot \frac{ft}{yr}$	background erosion rate

Analytical Solution to Pelnard-Considere for Rectangular Beachfill Project

$$M := \left[\frac{\sqrt{4 \cdot G \cdot t}}{l \cdot \sqrt{\pi}} \cdot \left[e^{-\left(\frac{l}{\sqrt{4 \cdot G \cdot t}}\right)^2} - 1 \right] + \text{erf}\left(\frac{l}{\sqrt{4G \cdot t}}\right) \right] - \frac{be \cdot t}{(Yo + Yb + Ya)}$$

fraction of initial fill remaining after time t

$M = 0.38$

$re := (1 - M) \cdot \frac{(Yo + Yb + Ya)}{t} = 21.638 \cdot \frac{ft}{yr}$ representative erosion rate

$de := re - be = 21.638 \cdot \frac{ft}{yr}$ diffusive erosion rate

Date: July 18, 2013
Analyst: Rob Hampson, Moffatt & Nichol
Client: U.S. Army Corps of Engineers
Project: Fire Island Interim
Analysis: Beachfill Diffusion - Davis Park - MIDU

Calculate fraction of beachfill volume remaining with Pelnard-Considere Equation for Rectangular Beachfill

Reference: Dean R. G., Dalyrmple R. A., 2002. "Coastal Processes with Engineering Applications," Cambridge University Press, New York, NY.)

Note: This analysis does not include beachfill tapers, the final analysis for the Fire Island Interim Project applied 6 degree berm tapers, which requires the trapazoidal beachfill solution to the Pelnard-Considere Equation. The trapzoidal beachfill solution was solved numerically in Matlab.

Definitions

$hc := 27 \cdot ft$	depth of closure (NGVD)
$B := 9.5 \cdot ft$	berm elevation (NGVD)
$G := 0.014 \cdot \frac{m^2}{s}$	alongshore diffusivity
$Yo := 20.4 \cdot ft$	initial cross-shore distance between MIDU design shoreline and natural shoreline
$Yb := 0 \cdot ft$	offset between MIDU baseline and evaluated baseline
$Ya := 48 \cdot ft$	advance nourishment width
$l := 4200 \cdot ft$	alongshore length of beachfill
$t := 4yr$	time after initial placement
$be := 0 \cdot \frac{ft}{yr}$	background erosion rate

Analytical Solution to Pelnard-Considere for Rectangular Beachfill Project

$$M := \left[\frac{\sqrt{4 \cdot G \cdot t}}{l \cdot \sqrt{\pi}} \cdot \left[e^{-\left(\frac{l}{\sqrt{4 \cdot G \cdot t}}\right)^2} - 1\right] + \mathrm{erf}\left(\frac{l}{\sqrt{4G \cdot t}}\right)\right] - \frac{be \cdot t}{(Yo + Yb + Ya)}$$

fraction of initial fill remaining after time t

$M = 0.262$

$$re := (1 - M) \cdot \frac{(Yo + Yb + Ya)}{t} = 12.626 \cdot \frac{ft}{yr}$$

representative erosion rate

$$de := re - be = 12.626 \cdot \frac{ft}{yr}$$

diffusive erosion rate

Date: July 18, 2013
Analyst: Rob Hampson, Moffatt & Nichol
Client: U.S. Army Corps of Engineers
Project: Fire Island Interim
Analysis: Beachfill Diffusion - Davis Park - MREI

Calculate fraction of beachfill volume remaining with Pelnard-Considere Equation for Rectangular Beachfill

Reference: Dean R. G., Dalyrmple R. A., 2002. "Coastal Processes with Engineering Applications," Cambridge University Press, New York, NY.)

Note: This analysis does not include beachfill tapers, the final analysis for the Fire Island Interim Project applied 6 degree berm tapers, which requires the trapazoidal beachfill solution to the Pelnard-Considere Equation. The trapzoidal beachfill solution was solved numerically in Matlab.

Definitions

$hc := 27 \cdot ft$	depth of closure (NGVD)
$B := 9.5 \cdot ft$	berm elevation (NGVD)
$G := 0.014 \cdot \frac{m^2}{s}$	alongshore diffusivity
$Yo := 20.4 \cdot ft$	initial cross-shore distance between MIDU design shoreline and natural shoreline
$Yb := 72.6 \cdot ft$	offset between MIDU baseline and evaluated baseline
$Ya := 145.7 \cdot ft$	advance nourishment width
$l := 4200 \cdot ft$	alongshore length of beachfill
$t := 4yr$	time after initial placement
$be := 0 \cdot \frac{ft}{yr}$	background erosion rate

Analytical Solution to Pelnard-Considere for Rectangular Beachfill Project

$$M := \left[\frac{\sqrt{4 \cdot G \cdot t}}{l \cdot \sqrt{\pi}} \cdot \left[e^{-\left(\frac{l}{\sqrt{4 \cdot G \cdot t}}\right)^2} - 1\right] + \operatorname{erf}\left(\frac{l}{\sqrt{4G \cdot t}}\right)\right] - \frac{be \cdot t}{(Yo + Yb + Ya)}$$

fraction of initial fill remaining after time t

$M = 0.262$

$re := (1 - M) \cdot \frac{(Yo + Yb + Ya)}{t} = 44.062 \cdot \frac{ft}{yr}$ representative erosion rate

$de := re - be = 44.062 \cdot \frac{ft}{yr}$ diffusive erosion rate

Date: July 18, 2013
Analyst: Rob Hampson, Moffatt & Nichol
Client: U.S. Army Corps of Engineers
Project: Fire Island Interim
Analysis: Beachfill Diffusion - Eastern Fire Island - MIDU

Calculate fraction of beachfill volume remaining with Pelnard-Considere Equation for Rectangular Beachfill

Reference: Dean R. G., Dalyrmple R. A., 2002. "Coastal Processes with Engineering Applications," Cambridge University Press, New York, NY.)

Note: This analysis does not include beachfill tapers, the final analysis for the Fire Island Interim Project applied 6 degree berm tapers, which requires the trapazoidal beachfill solution to the Pelnard-Considere Equation. The trapzoidal beachfill solution was solved numerically in Matlab.

Definitions

$hc := 27 \cdot ft$	depth of closure (NGVD)
$B := 9.5 \cdot ft$	berm elevation (NGVD)
$G := 0.014 \cdot \frac{m^2}{s}$	alongshore diffusivity
$Yo := 6.8 \cdot ft$	initial cross-shore distance between MIDU design shoreline and natural shoreline
$Yb := 0 \cdot ft$	offset between MIDU baseline and evaluated baseline
$Ya := 8 \cdot ft$	advance nourishment width
$l := 19400 \cdot ft$	alongshore length of beachfill
$t := 4yr$	time after initial placement
$be := 1 \cdot \frac{ft}{yr}$	background erosion rate

Analytical Solution to Pelnard-Considere for Rectangular Beachfill Project

$$M := \left[\frac{\sqrt{4 \cdot G \cdot t}}{l \cdot \sqrt{\pi}} \cdot \left[e^{-\left(\frac{l}{\sqrt{4 \cdot G \cdot t}}\right)^2} - 1\right] + \operatorname{erf}\left(\frac{l}{\sqrt{4G \cdot t}}\right)\right] - \frac{be \cdot t}{(Yo + Yb + Ya)}$$

fraction of initial fill remaining after time t

$$M = 0.476$$

$$re := (1 - M) \cdot \frac{(Yo + Yb + Ya)}{t} = 1.938 \cdot \frac{ft}{yr}$$

representative erosion rate

$$de := re - be = 0.938 \cdot \frac{ft}{yr}$$

diffusive erosion rate